

# Quantum signature scheme with single photons

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## Abstract

Quantum digital signature combines quantum theory with classical digital signature. The main goal of this field is to take advantage of quantum effects to provide unconditionally secure signature. We present a quantum signature scheme with message recovery without using entangle effect. The most important property of the proposed scheme is that it is not necessary for the scheme to use Greenberger-Horne-Zeilinger states. The present scheme utilizes single photons to achieve the aim of signature and verification. The security of the scheme relies on the quantum one-time pad and quantum key distribution. The efficiency analysis shows that the proposed scheme is an efficient scheme.

**keywords:** Quantum signature, Quantum one-time pads, Quantum key distribution

## 1 Introduction

Quantum cryptography is a cryptographic system using quantum effects to provide unconditionally secure information exchange. Many advances have been made in quantum cryptography in recent years, including quantum key distribution (QKD)[3, 4, 5], quantum secret sharing[6], quantum authentication and quantum signature[1, 2, 7, 8, 9, 10]. Classical digital signature is the basis of realizing identity authentication, data integrity protection and non-repudiation services. Because classical digital signature is not unconditionally secure, some quantum signature schemes are proposed in recent years[1, 2, 8, 9, 10]. Quantum digital signature combines quantum theory with classical digital signature and utilizes quantum effects to achieve unconditional security.

Gottesman and Chuang proposed a quantum signature scheme based on quantum one-way functions and quantum Swap-test[8]. However, their scheme is an inefficient scheme. Zeng and Christoph proposed an arbitrated quantum signature scheme whose security based on the correlation of the Greenberger-Horne-Zeilinger (GHZ) triplet states and the use of quantum one-time pad[1]. Lee et al. proposed two quantum signature schemes with message recovery[2]. Lee's schemes also use GHZ triplet states, qubit operations and quantum one-time pad to generate and verify the signature. Lü and Feng presented two quantum signature schemes, one scheme based on quantum one-way functions and the other based on GHZ triplet states and quantum stabilizer codes[9, 10].

Most of the proposed quantum signature schemes use entangle effect to achieve the aim of signature and verification. In this paper, we present a quantum signature scheme with message recovery without using entangle effect. The communication parties utilize single photons to perform signature and verification with the help of an arbitrator. The security of our scheme relies on quantum one-time pad and quantum key distribution proved as unconditionally secure[11, 12]. Because our scheme does not need to distribute the GHZ particles and only needs von Neumann measurement, our scheme provides higher efficiency.

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## 2 Description of our quantum signature scheme

Our scheme is composed of three parts: initial phase, signature phase and verification phase, involving three participants, the signatory Alice, the receiver Bob and the arbitrator. In the initial phase, the arbitrator distributes secret information to each Alice and Bob. In the signature phase, Alice generates her signature in association with the message qubits, the secret information and qubit operations. In the verification phase, Bob verifies Alice's signature with the help of the arbitrator.

Both QKD and quantum one-time pad play important roles in our scheme. We use QKD protocol, such as BB84 or B92 protocol to distribute the secret information. The encryption algorithm used in our scheme is quantum one-time pad described in [13]. We select a set of  $M$  unitary operations  $\{U_k\}$ ,  $k = 1, \dots, M$ . The key  $k$  is chosen with probability  $p_k$  and the message state is  $\rho$ . Then we obtain the cipher state  $\rho_c$ , where  $\rho_c = \sum_k p_k U_k \rho U_k^\dagger = \frac{1}{2^n} I$ .

### 2.1 Initial Phase

Alice and Bob share secret keys  $K_a$ ,  $K_b$  with the arbitrator, respectively. These keys are assumed to be distributed via QKD protocol. The arbitrator generates two random secret bit strings  $A$  and  $B$ , where  $A = \{A_1, A_2, \dots, A_n\}$ ,  $B = \{B_1, B_2, \dots, B_n\}$ ,  $A_i, B_i (i = 1, 2, \dots, n) \in \{0, 1\}$ , as authentication keys and distributes the two strings to each Alice and Bob via QKD protocol.

### 2.2 Signature Phase

In the signature phase, Alice signs her message  $|P\rangle$  and obtains her signature  $|S\rangle$ .

Step1: At the beginning, Alice prepares a string of message qubits  $|P\rangle = \{|p_1\rangle, |p_2\rangle, \dots, |p_n\rangle\}$ , where any qubit  $|p_i\rangle$  ( $i = 1, 2, \dots, n$ ) in  $|P\rangle$  is one of the two eigenstates  $|0\rangle, |1\rangle$ .  $|P\rangle$  may be presented by classical bits. We represent the message string as qubits so that the signature can be transmitted through quantum channel.

Step2: In this step, Alice transforms the message qubits  $|P\rangle$  into state  $|M\rangle$  according to  $A$ . If  $A_i = 0$ , Alice does nothing to  $|p_i\rangle$ , otherwise she applies bit flip gate

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

to  $|p_i\rangle$ . Then after Alice's qubit operations, she obtains

$$|M\rangle = \{|m_1\rangle, |m_2\rangle, \dots, |m_n\rangle\} = \{|A_1 \oplus p_1\rangle, |A_2 \oplus p_2\rangle, \dots, |A_n \oplus p_n\rangle\}.$$

Step3: In this step, Alice transforms the message qubits  $|P\rangle$  into state  $|R\rangle$  according to  $K_a$ . If  $K_a^i = 0$ , Alice uses rectilinear measurement basis  $\{|\rightarrow\rangle, |\uparrow\rangle\}$ . If  $K_a^i = 1$ , Alice uses diagonal measurement basis  $\{|\nearrow\rangle, |\searrow\rangle\}$ . Then  $|0\rangle, |1\rangle$  of the message qubits  $|P\rangle$  are represented by  $|\rightarrow\rangle$  and  $|\uparrow\rangle$  in the rectilinear measurement basis and by  $|\nearrow\rangle$  and  $|\searrow\rangle$  in the diagonal measurement basis, respectively. Alice obtains

$$|R\rangle = M_{K_a}|P\rangle = \{|r_1\rangle, |r_2\rangle, \dots, |r_n\rangle\},$$

where  $|r_i\rangle = M_{K_a}^i |p_i\rangle (i = 1, 2, \dots, n)$ .

Step4: Alice obtains the quantum signature  $|S\rangle$  for the message qubits  $|P\rangle$  by encrypting  $|M\rangle$  and the secret qubits  $|R\rangle$  with the key  $K_a$ .

$$|S\rangle = E_{K_a}\{|M\rangle, |R\rangle\}.$$

Step5: Alice sends the signature  $|S\rangle$  to Bob through a quantum channel.

### 2.3 Verification phase

In the verification phase, Bob verifies Alice's signature  $|S\rangle$  with the help of the arbitrator.

Step1: After receiving the signature  $|S\rangle$ , Bob encrypts  $|S\rangle$  and  $B$  with the key  $K_b$  and obtains

$$|N\rangle = E_{K_b}\{|S\rangle, B\rangle.$$

Then he sends  $|N\rangle$  to the arbitrator.

Step2: The arbitrator decrypts  $|N\rangle$  with the key  $K_b$  and obtains  $|S\rangle$ ,  $B'$ . Then he decrypts  $|S\rangle$  with the key  $K_a$  and obtains  $|M\rangle$ ,  $|R\rangle$ .

Step3: The arbitrator recovers the message  $|P\rangle$  with  $|M\rangle$  and  $A$  for

$$|P\rangle = |A \oplus M\rangle = \{|A_1 \oplus m_1\rangle, |A_2 \oplus m_2\rangle, \dots, |A_n \oplus m_n\rangle\}.$$

The arbitrator then generates  $|R'_a\rangle$  using  $|P\rangle$  and  $K_a$  according to the methods of the third step in the signature phase. He compares  $|R'_a\rangle$  with  $|R\rangle$  and generates a verification parameter  $\gamma$ . If  $|R'_a\rangle = |R\rangle$ , he sets  $\gamma = 0$ , otherwise  $\gamma = 1$ . The arbitrator also compares  $B'$  with  $B$  and generates parameter  $\xi$ . If  $B' = B$ , he sets  $\xi = 0$ , otherwise  $\xi = 1$ . The arbitrator transforms the message qubits  $|P\rangle$  into  $|R'_b\rangle$ , where  $|R'_b\rangle = M_{K_b}|P\rangle$ , using  $K_b$ . He also generates  $|U\rangle = |B \oplus P\rangle$  using  $B$  and  $|P\rangle$ .

Step4: The arbitrator encrypts  $\gamma$ ,  $\xi$ ,  $|U\rangle$ ,  $|R'_b\rangle$  and  $|S\rangle$  using the key  $K_b$  and obtains

$$|V\rangle = E_{K_b}\{\gamma, \xi, |U\rangle, |R'_b\rangle, |S\rangle\}.$$

Then he sends  $|V\rangle$  to Bob.

Step5: Bob decrypts  $|V\rangle$  and obtains  $\gamma$ ,  $\xi$ ,  $|U\rangle$ ,  $|R'_b\rangle$ ,  $|S\rangle$ . If  $\gamma = \xi = 0$ , he then recovers the message qubits by  $|P\rangle = |B \oplus U\rangle$ . Bob generates  $|R'\rangle$  using  $|P\rangle$  and  $K_b$ , and compares it with  $|R'_b\rangle$ . If  $|R'\rangle = |R'_b\rangle$ , he accepts the message qubits  $|P\rangle$  and the signature  $|S\rangle$ , otherwise he should reject it.

## 3 Security analysis

The security of signature schemes requires that the signatory can not disavow her signature and the signature can not be forged. We demonstrate that our scheme is unconditionally secure as follows.

### 3.1 Impossibility of forgery

**Theorem 1.** *If other entities forge Alice's signature, their cheating will be detected with a probability  $P_r \geq 1 - \frac{1}{2^{|K_a|+|A|}}$ .*

**Proof.** The signature is generated by encrypting the state  $|R\rangle$  and  $|M\rangle$  with  $K_a$ , secretly kept by Alice and the arbitrator.  $K_a$  is distributed via QKD protocol proved as unconditionally secure.  $|R\rangle$ ,  $|M\rangle$  are states, into which Alice transforms the message qubits  $|P\rangle$  according to  $K_a$  and  $A$ , respectively. Firstly, we assume that Bob is dishonest and tries to forge Alice's signature. However,  $|R\rangle$ ,  $|M\rangle$  and  $K_a$  are secret for him. Secondly, we assume an attacker, Eve tries to forge Alice's signature. The public parameters of our scheme are  $|S\rangle$ ,  $|N\rangle$  and  $|V\rangle$ , where  $|S\rangle = E_{K_a}\{|M\rangle, |R\rangle\}$ ,  $|N\rangle = E_{K_b}\{|S\rangle, B\rangle$ ,  $|V\rangle = E_{K_b}\{\gamma, \xi, |U\rangle, |R'_b\rangle, |S\rangle\}$  and they do not offer any information of the secret keys. The encryption algorithm is quantum one-time pad proved as unconditionally secure. If Bob or Eve randomly selects the two strings  $K'_a$  and  $A'$  to execute the scheme, their cheating will be detected by the arbitrator with a probability larger than  $1 - \frac{1}{2^{|K_a|+|A|}}$ , where  $|K_a|$  and  $|A|$  denote the length of the  $K_a$  and  $A$ , respectively.

### 3.2 Impossibility of disavowal

**Theorem 2.** *If Alice denies her signature, the arbitrator can judge whether Alice has disavowed her signature.*

**Proof.** Because the signature  $|S\rangle$  contains Alice's key  $K_a$ , Alice can not disavow her signature. If Alice disavows her signature, Bob only need to send the signature  $|S\rangle$  to the arbitrator and the arbitrator will judge whether Alice has disavowed her signature. If the signature contains Alice's secret keys, this signature has been carried by Alice, otherwise, the signature has been forged by other entities. On the other hand, Bob can not deny his receiving of Alice's signature because he needs the help of the arbitrator in the verification phase.

## 4 Efficiency analysis

We simply define the efficiency of quantum signature schemes as the formula  $\eta = B_s/(Q_t + B_t)$ , where  $B_s$  is the number of message qubits signed,  $Q_t$  and  $B_t$  are each the number of qubit transmitted and classical bit exchanged in the scheme. In terms of the formula, the efficiency of our scheme, Lee's scheme and Zeng's scheme is each 11%, 11% and 9%. However, the formula is just used to simply analyze the efficiency of quantum signature schemes. In practice, we should also consider the complexity of realizing a scheme, involving the distribution of initial secret information, measurement methods, encryption algorithm, etc.

Although an arbitrator is not necessary in Gottesman's scheme, it requires a trusted key distribution center, which has authenticated links to all three participators. Moreover,  $M$  keys are used to sign each bit message in Gottesman's scheme. Zeng's scheme requires a joint measurement on each message qubit and GHZ particle to generate a four-particle entangled state and  $n$  Bell measurement to disentangle message qubits and GHZ particles. In Lee's scheme, the signatory and the verifier should also measure their GHZ particles and compare qubits strings in the verification phase. Lü's scheme is relatively complicated, because the secret keys and the syndromes of quantum stabilizer codes are each used to encrypt and encode the quantum states. Our scheme does not need to use the GHZ triplet states, so it reduces the process of distributing and measuring the GHZ particles. In the verification phase, we only need to use von Neumann measurement to verify the qubits strings. Compared with these quantum signature schemes, our scheme provides higher efficiency.

## 5 Conclusion

In this paper, we propose a quantum signature scheme with single photons. In the initial phase of our scheme, secret keys and authentication keys are distributed to each participator via QKD protocol. In the signature phase, Alice generates her signature by encrypting the transformed message qubits with the secret keys. In the verification phase, Bob verifies Alice's signature with the help of the arbitrator. The security analysis showed that our scheme is unconditionally secure. We only need to utilize QKD, quantum one-time pad, qubit operations and von Neumann measurement to realize our scheme, so the scheme is simple and practicable. Many potential improvements of quantum signature remain largely unexplored. The properties of quantum should be fully used to design more practicable quantum signature schemes.

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